PHYS5150 — PLASMA PHYSICS

LECTURE 19 - AMBIPOLAR DIFFUSION

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Spring 2023

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1 PLASMA COLLISION FREQUENCY

Let us quickly return to the mean free path length λ_{mfp}, which we derived in lecture 16:

$$\lambda_{mfp} = \frac{1}{\pi n b_0^2 \ln \Lambda}.$$

Recall that λ_{mfp} is based on accumulated small angle collisions. Now, in analogy to solid sphere collisions we define the *collision time* τ_c as the time to go one λ, i.e.

$$\tau_c = \frac{\lambda_{mfp}}{v} = \lambda_{mfp}\sqrt{\frac{m}{2kT}},$$

which allows us to determine the collision rate $\nu_c = 1/\tau_c$

$$\nu_c = \pi n b_0^2 \sqrt{\frac{2kT}{m}} \ln \Lambda$$

resulting from accumulated small angle collisions. Recall that collisions are driving diffusion, which implies that the *diffusion coefficient* is

$$D = \frac{kT}{m\nu}.$$

We first consider diffusion in a plasma composed of electrons and ions in absence of a magnetic field.

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2 AMBIPOLAR DIFFUSION (B=0)

We start with the continuity equation

$$\frac{\partial n}{\partial t} + \nabla(\mathbf{u}n) = \frac{\partial n}{\partial t} + \nabla\mathbf{\Gamma} = 0.$$

Even in the presence of diffusion, the plasma needs to maintain charge neutrality, i.e.

$$n_i = n_e.$$

Hence,

$$\nabla\mathbf{\Gamma}_i = \nabla\mathbf{\Gamma}_e,$$

implying that

$$\mathbf{\Gamma}_i = \mathbf{\Gamma}_e,$$

or in other words, the ions and electrons must flow together to maintain neutral charge density.

Now, if the diffusion rates of the ions and electrons are not identical, then an electric field will built up until the electron flux

$$\mathbf{\Gamma}_e = \mu_e n\mathbf{E} - D_e \nabla n$$

and ion flux

$$\mathbf{\Gamma}_i = \mu_i n\mathbf{E} - D_i \nabla n$$

are equal again

$$\mathbf{\Gamma}_i - \mathbf{\Gamma}_e = 0 = n(\mu_i + \mu_e)\mathbf{E} - (D_i - D_e)\nabla n,$$

or

$$\mathbf{E} = \left(\frac{D_i - D_e}{\mu_i + \mu_i}\right)\frac{\nabla n}{n},$$

where

$$\mu = \frac{|q|}{m\nu}$$

is the *mobility*.

Difference in diffusion rates dictates higher E fields to hold populations together. Similarly, higher mobility increases effectiveness of E field, so it requires weaker electric fields.

We now compute the flux $\mathbf{\Gamma} = \mathbf{\Gamma}_i$:

$$\begin{aligned}
\mathbf{\Gamma} = \mu_i n\mathbf{E} - D_i\nabla n &= \mu_i n\left(\frac{D_i - D_e}{\mu_i + \mu_i}\right)\frac{\nabla n}{n} - D_i\nabla n \\
&= \left(\frac{\mu_i D_i - \mu_i D_e - \mu_i D_i - \mu_e D_i}{\mu_i + \mu_e}\right)\nabla n \\
&= -\left(\frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}\right)\nabla n = -D_A\nabla n
\end{aligned}$$

where

$$\boxed{D_A = \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}}$$

is the *ambipolar diffusion coefficient*, which describes the diffusion of electrons and ions together. Because the electron mobility exceeds the ion mobility by far,

$$\begin{aligned}
D_A &\approx \frac{\mu_i}{\mu_e}D_e + D_i \\
&\approx D_i\left(1 + \frac{T_e}{T_i}\right),
\end{aligned}$$

which is $= D_i$ for $T_e = T_i$. Electrons diffuse faster, resulting in an E field which holds them back and pulls the ions faster. The net effect is diffusion at twice the ion rate.